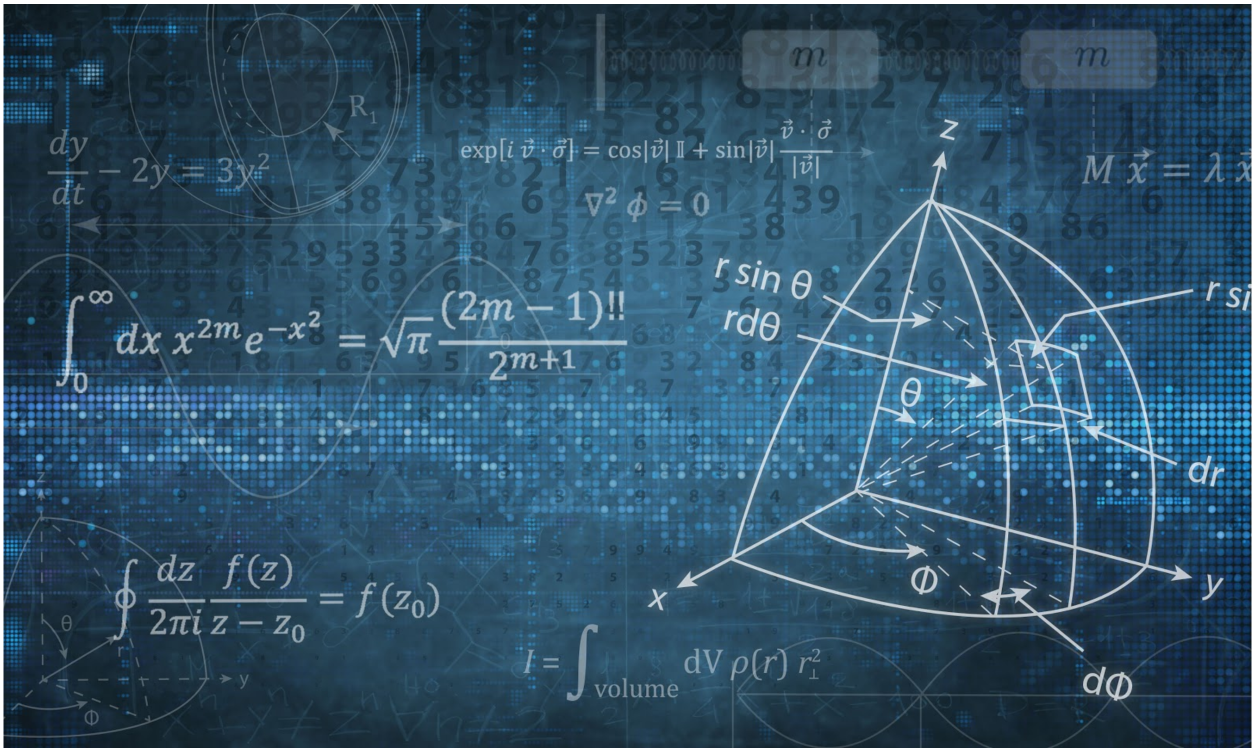

Mathematical and Computational Methods

Course Syllabus

An instructor-led course by GeorgetownX (PHYS-155)

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Mathematical and Computational Methods

Physicists use math all of the time in nearly everything that they work on. Hence, it is critical that you become efficient in being able to use more advanced math to enable you to work on more advanced physics courses. The goal of this class is to transform you from a math technician to a math practitioner. Mathematicians take this one step further and actually create new math. We will not focus on how to do that at all in this class.

NOTE: The course will be available through June 14, 2021. No certificates will be issued for work completed after June 14, 2021.

LEAD FACULTY

James Freericks
Professor and McDevitt Chair
Georgetown University

James Freericks is a Professor of Physics who works in theoretical and computational physics focused on quantum mechanical phenomena. He has published over 200 peer-reviewed articles, an award-winning textbook, and has taught quantum mechanics to nonscientists since 1995. He is a Fellow of the American Physical Society and the American Association for the Advancement of Science. He also is the Divisional Councilor of the American Physical Society's Division of Computational Physics.

FACULTY SUPPORT TEAM

You can contact the course support team at gux@georgetown.edu.

WHAT IS THE COURSE ABOUT?

In physics, we use advanced math to describe many, many different things. We need to become practitioners of math. This is more than just knowing the technical details for how to perform mathematical manipulations. It is about properly understanding the details for the how and the why of the math. It is understanding the "big picture" for how mathematics is used to describe the physical world.

I describe this as making the transition from a technician to a practitioner. A technician is someone who learns a particular skill without thinking carefully about when it applies, how one applies it in a novel context, and what its limitations might be. Practitioners, know all that and more. They are experts in knowing how to use math as a tool to help elaborate the properties of the physical world. All physicists are practitioners of math. This course is the first step in your journey towards this goal.

We do not cover all the math you will need in your career. Just a small fraction. But by learning how to learn about math in this fashion, where we connect the math ideas to physical principles, you will be able to learn whatever more math you need on your own, or in the context of an upper-level physics class. Because the level of coverage of math is different between being a practitioner and creating new knowledge as a mathematician, we can and do cover a lot of ground in this class. This is simply because physicists need to know a lot.

The specific topics we cover is single-variable calculus, multivariable calculus, vector calculus theorems, complex numbers and complex variables (including the residue theorem), linear algebra, first-order linear and nonlinear differential equations, and second-order differential equations with constant coefficients.

WHAT DOES THE COURSE CONTENT INCLUDE?

The course is broken into a number of different themes and then within each theme, we have a number of specific topics that we cover.

In a broad brush, we provide a calculus review followed by the integral integral theorems of multivariable calculus. Then we have a short unit on complex numbers, which also develops the calculus of residues. Linear algebra follows, then differential equations and finally Fourier series.

Course Section Outline

Calculus Review

1. The Greeks and the development of the limit
2. The logarithm and the revolution of numerical computation from tables
3. The concept of the derivative and antiderivative
4. How to integrate
5. Multivariable integrals
6. Frullani and Gaussian integrals
7. Feynman integration

Vector Calculus

1. Vector fields
2. The divergence theorem

3. Stokes theorem
4. Gradient
5. Laplace's equation

Complex Analysis
1. Complex numbers
2. Analytic functions
3. Cauchy's theorem
4. Calculus of residues

Linear Algebra
1. Matrix multiplication
2. The determinant
3. Matrix inverses
4. Eigenvalues and eigenvectors
5. Abstract vector spaces with inner products

Differential Equations
1. First-order linear differential equations
2. First-order nonlinear differential equations
3. Differential equations with constant coefficients
4. Variation of parameters
5. Method of undetermined coefficients
6. Frenet-Serret apparatus

Fourier Series
1. Dirichlet theorem and Fourier summation formula
2. Poisson's theorem

WHAT WILL I LEARN IN THE COURSE?

The underlying colloquial goal is to transform from a technician to a practitioner in math. But this is vague from a learning design standpoint. So the precise learning goals are as follows:

Specific learning goals for the course
- Be able to apply techniques of calculus (learned in the first three semesters of a calculus sequence) to solve problems that arise in physics.
- Derive and use the geometric series in calculations.
- Manipulate power series expressions and employ them in physics contexts.

- Calculate Taylor polynomials/series of common functions and use them in approximating functions
- Follow the development for how one integrates polynomials, rational functions of polynomials, square roots of quadratics, rational functions with square roots of quadratics, and why the procedure cannot solve integrals with square roots of quartics.
- Solve integrals via parametric methods (differentiating under the integral sign) including techniques for introducing the parameter into the integrand
- Set up and integrate multidimensional integrals with variable mass density and for moments of inertia.
- Solve problems in multivariable integrals via the different integral theorems
- Solve Laplace's equation for simple geometries
- Compute the curl or divergence of a vector field and the gradient of a scalar function
- Solve complex numbers arithmetic problems
- Use calculus of residues to solve integrals
- Use row reduction to solve simultaneous linear equations
- Calculate the determinant, inverse and eigenvalues of matrices
- Construct abstract vector spaces using the definitions of a vector space and generalized inner products
- Solve any first-order linear differential equation
- Solve the six classes of nonlinear first-order differential equations
- Solve higher-order differential equations with constant coefficients (homogeneous and inhomogeneous)
- Construct representations of curves in three dimensions using the Frenet-Serret apparatus

WHAT SHOULD WE EXPECT FROM EACH OTHER?

What You Can Expect From Professor Freericks and the Course Team

The instructor will regularly participate in discussion boards to provide content clarification, guidance, and support.

You can also email us with important content-related questions at gux@georgetown.edu.

What You Can Expect From edX

In the event of a technical problem, you can go to edX Learner Help Center. At the Learner Help Center, you can also:

- Report a problem
- Make a suggestion
- Ask a question

You may also contact technical@edx.org directly to report bugs.

What We Can Expect From You

You should expect to spend around seven to ten (7-10) hours per week to review the written content, watch lectures, and work through practice questions and other activities, including the discussion boards.

In each course section, we have included activities to support you in reaching the specified learning objectives for that section. They include:

- **Video lectures**: The video lectures are all in the Khan academy style of voice-over powerpoints. We use PowerPoint rather than writing on a tablet because my handwriting is awful, and PowerPoints allow for much more extra features such as animations. Most lectures are about 10 mins.
- **Lecture problems**: We do lecture problems in class just after the lecture is watched by all of you prior to coming to class. You will work on the problems in small groups and get assistance from instructors when you need it.
- **Laboratories**: Once a week, we have a laboratory. This involves a demonstration of lab equipment (brought to you by video) or specialized instruction that applies the ideas to a physics application.
- **Problem sets**: The class has weekly problem sets. Every problem set has a one week grace period built-in. The problem sets are due two weeks after they appear in the class. The lowest homework score is dropped from your grade.
- **Problem of the week**: There is one problem (in either the laboratory or the lecture problems) that is particularly challenging. We include a video showing the detailed solution to each problem of the week. Please watch the video only after you have tried to solve the problem. Give it a good try before looking at the solution.
- **Exams**: There are three midterms and a final in the class. The midterms are all scheduled for lab days. The exam itself is given in a timed period of your choosing during the exam day.

Your final grade will be determined as follows:

- Lecture problems and Laboratories 0%
- Problem sets 45%

- Each Midterm 10% for a total of 30%
- Final Exam 25%

Your final exam must be completed by the end of the self-paced course, which is currently scheduled for June 14, 2021. The last chance to register for the Verified Track, to earn a course certificate, is June 4, 2021, but in order to earn a certificate, you will need to register earlier as all work has deadlines.

All activities included in the course are designed to help you gauge your learning as a result of your interaction with the course content. Instructions on how to complete the activities are included within each course section.

Netiquette Guidelines

Please be respectful
To promote the best educational experience possible, we ask each student to respect the opinions and thoughts of other students and be courteous in the way that you choose to express yourself. Informed debate should never give way to insult, rudeness, or anything that might detract from the learning process. PHYS-155 students should be respectful and considerate of all opinions.

In order for us to have meaningful discussions, we must learn to understand what others are saying and be open-minded about others' opinions. If you want to persuade someone to see things differently, it is much more effective to do so in a ***polite, non-threatening way*** rather than to do so antagonistically. Everyone has insights to offer based on his/her experiences, and we can all learn from each other. Civility is essential: Our course staff can, and will, remove students from the class who detract from the learning process with insulting comments on the course-wide discussion boards.

Look before you write
Prior to posting a question or comment on the discussion board, the Georgetown course team asks that you look to see if any of your classmates have the same question. Upvote questions that are similar to your own or that are also of interest to you, instead of starting a new thread. This will greatly help our course team best monitor the discussions and bring important questions to Professor Freerick's attention.

Use the discussion board for course-related posts only
Although we encourage students to get to know each other, please use the discussion board for course content conversations only.

Properly and promptly notify us of technical issues
Although we do not predict technical issues, they can and may happen. To make sure these receive prompt attention, post details about any technical issues directly on the "Technical"

discussion thread or email technical@edx.org directly. You may reach us with course-related content questions at gux@georgetown.edu.

Academic Integrity

Observe edX and GeorgetownX's honor policies
Science is a collaborative venture, and you will find being able to discuss practice questions with peers will often help you to learn the material. This practice is encouraged on the discussion board, however, one is not allowed to post answers—instead, focus on asking for help on what you do not understand and to seek clarification; problematic posts may be edited by the instructor. It is expected that all work that you submit for this course is your own. In particular, the exams are not to be discussed on the discussion boards or elsewhere. Violations of the honor policy undermine the purpose of education and the academic integrity of the course. We expect that all work submitted will be a reflection of your own original work and thoughts and that you will abide by the honor policy to allow others to also experience the course and learn from it. GeorgetownX faculty and staff expect all members of the community to strive for excellence in scholarship and character. This is part of Georgetown's pursuit of Cura Personalis or educating the whole person.

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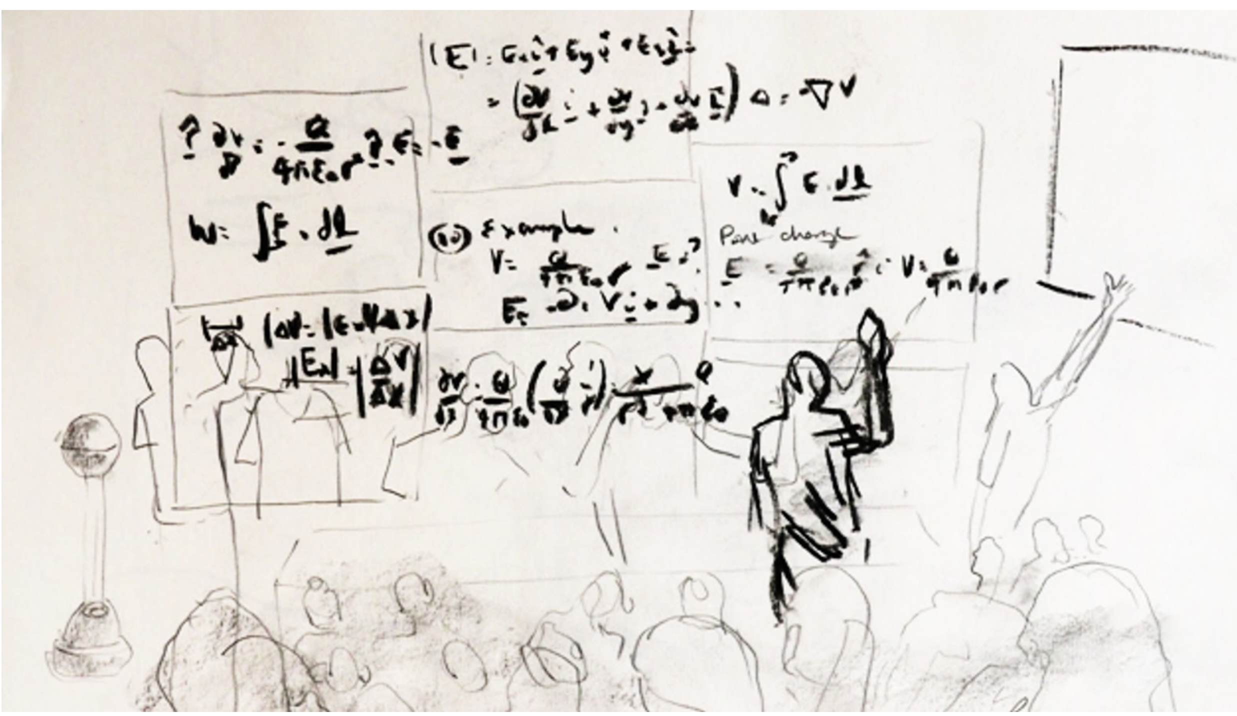

Image of Peter Turk's lecture at Imperial College courtesy of Geraldine Cox